

Bond Duration and Convexity

http://spreadsheetml.com/finance/bondduration_convexity.shtml

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ConnectCode's Financial Modeling Templates

Have you thought about how many times you use or reuse your financial models? Everyday, day after day, model after model and project after project. We definitely have. That is why we build all our financial templates to be reusable, customizable and easy to understand. We also test our templates with different scenarios vigorously, so that you know you can be assured of their accuracy and quality and that you can save significant amount of time by reusing them. We have also provided comprehensive documentation on the templates so that you do not need to guess or figure out how we implemented the models.

All our template models are only in black and white color. We believe this is how a professional financial template should look like and also that this is the easiest way for you to understand and use the templates. All the input fields are marked with the '' symbol for you to identify them easily.*

Whether you are a financial analyst, investment banker or accounting personnel. Or whether you are a student aspiring to join the finance world or an entrepreneur needing to understand finance, we hope that you will find this package useful as we have spent our best effort and a lot of time in developing them.

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1. Bond Duration and Convexity

1.1 Background

If an investor is given a choice of two 10-year bonds to choose from, one with a 10 percent coupon rate and the other with a 5 percent coupon rate. Assuming the risk of default is the same for the two bonds, the investor will likely choose the one with the higher coupon rate. From a returns perspective, the 10 percent coupon allows the investor to recoup his or her investment in a shorter time frame. The shorter time frame is considered good as it allows the investor to be less exposed to a risk known as the Interest Rate Risk.

1.2 Interest Rate Risk

Interest rate may go up or down at different points in time for different reasons. For example, during a recession, interest rate will be cut to kick-start the economy. When the economy recovers and exhibits behaviour of inflation, the interest rate may increase. It is known that when interest rate goes up, it causes a decrease in the price of a bond. The reverse where interest rate goes down causing an increase in bond price is also true. The risk of the decrease in the price of a bond due to an increase in the interest rate is known as the Interest Rate Risk.

One of the factors affecting the Interest Rate Risk is the maturity of a bond. With a longer time frame, the bond is more likely to subject to change in price due to a interest rate change. The other factor affecting the Interest Rate Risk is the Coupon Rate. A higher coupon rate allows the price of the bond to be recovered in a shorter time frame and thus expose an investor to less Interest Rate Risk.

1.3 Reinvestment Rate Risk

We might think that since a decrease in interest rates lead to an increase in Bond Price, it will not in anyway adversely affect a bond's investor. However, a decrease in interest rate will lead to a decrease in future income of the bond investor. For example, the coupon amount received by an investor can only be reinvested in lower yielding bonds. This risk of a decrease in income due to a decrease in interest rates is known as the Reinvestment Rate Risk.

1.4 Bond Duration

In 1938, Federick Macaulay conceived the idea of a measurement call the Duration to measure the Interest Rate Risk. It combines the maturity of a bond and the coupon rate and can be thought of as how long it takes for the price of a bond to be recovered.

1.4.1 Macaulay Duration

Macaulay Duration can be calculated as follows:

Duration = $\text{Sum} ((\text{Present Value of Cash Flow at Time } t * t) / \text{Bond Market Price})$

- t - the time period of the cash flow. If the number of years to maturity is 10 then t is 1 to 10.
- Present Value of Cash Flow at Time t - The present value of the cash flow is discounted using the Yield to Maturity.
- Bond Market Price - The present value of all cash flows of the bond.

1.4.2 Modified Duration

One of the interesting side effects of Macaulay Duration is that it can be adjusted to approximate the interest rate sensitivity of a bond. This is the approximation of the percentage change in the

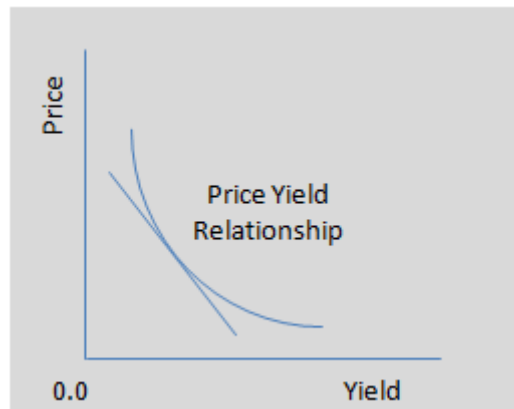
price of the bond to the percentage change in yield. The basic assumptions are that the bond is not tied to any Options and changes in yields are small.

Modified Duration is calculated as follows:

Modified Duration = Macaulay Duration / (1 + (Current Yield to Maturity/Number of Payments in a year))

1.5 Convexity

Modified Duration can be used to approximate the price change of a bond in a linear manner as shown in the formula above. In fact, the price and change in interest rate change relationship is not exactly linear. The relationship is actually curvilinear. The diagram below shows the approximation using Modified Duration as a straight line and the actual price-interest rate change relationship as a curve. It is important to note that the degree of the "curvature" of the curve is known as the convexity.



1.5.1 How to use the Bond Convexity

Bond Convexity is defined formally as the degree to which the duration changes when the yield to maturity changes. It can be used to account for the inaccuracies of the Modified Duration approximation. On top of that, if we assume two bonds will provide the same duration and yield then the bond with the greater convexity will be less affected by interest rate change. This can be easily visualized from the diagram above where the greater the "curvature", the lesser the price drop when interest rate increase.

1.6 Bond Duration Spreadsheet

The "Bond Duration" worksheet allows you to calculate the Duration of a bond quickly and easily.

	A	B	C	D	E	F	G
1	Bond Duration and Convexity						
2							
3	Copyright © 2009, ConnectCode						
4	Coupon Payment Frequency (pf)*				Semi-Annually		
5	Face Value (F)*				\$1,000.00		
6	Number of Periods to Maturity (N)*				10.00		
7	Coupon Rate (I)*				5.00%		
8	Yield to Maturity (Y)*				10.00%		
9	Discount Rate per period (r)				5.00%		
10	Duration				4.41		
11	Modified Duration				4.20		
12	Convexity				23.11		
13							
14	Periods (t)	Coupon Cash Flows	Face Value	Total Cash Flows	PV of Cash Flows	Proportion of Bond's value * Time	(PV*(t^2+t))
15	1	\$25.00	\$0.00	\$25.00	\$23.81	0.014752666	17.85714286
16	2	\$25.00	\$0.00	\$25.00	\$22.68	0.028100317	45.35147392
17	3	\$25.00	\$0.00	\$25.00	\$21.60	0.04014331	80.98477486
18	4	\$25.00	\$0.00	\$25.00	\$20.57	0.050975632	123.4053712
19	5	\$25.00	\$0.00	\$25.00	\$19.59	0.060685276	171.3963489
20	6	\$25.00	\$0.00	\$25.00	\$18.66	0.069354601	223.864619
21	7	\$25.00	\$0.00	\$25.00	\$17.77	0.077060668	279.8307737
22	8	\$25.00	\$0.00	\$25.00	\$16.92	0.083875557	338.419681
23	9	\$25.00	\$0.00	\$25.00	\$16.12	0.089866668	398.8517669
24	10	\$25.00	\$1,000.00	\$1,025.00	\$629.26	3.89897712	18877.83255

1.6.1 Input Values

- Coupon Payment Frequency (pf) – This field indicates whether the coupon is paid annually or semi-annually. The coupon rate is typically stated in an annual percentage. Thus if a coupon is paid out semi-annually, the coupon payments is equivalent to :

(Coupon Rate / 2) * Face Value of the Bond
- Face Value (F) - The principal or loan amount of the bond to be repaid at the end of the maturity period.
- Number of Periods to Maturity (N) - This field is related to the Coupon Payment Frequency. If Coupon Payment Frequency is set to Semi-Annually, Number of Periods means number of Half-year period. If Coupon Payment Frequency is set to Annually, then Number of Periods means number of One-year period. This field is used in the calculation of the Bond's Yield to Maturity.
- Coupon Rate (I) - This is the stated annual interest rate payments for a Bond. This interest rate multiply with the Face value gives the periodic coupon payments.
- Bond Price (v) - The current price of the bond in the market. Bond prices fluctuates due to changes in interest rates and the price that the bond is purchased affects the Yield to Maturity.
- Yield to Maturity Type – This Bond Valuation spreadsheet distinguishes between the Annual Percentage Rate and the Effective Annual Rate. When people talk about yield to maturity,

they typically refer to the Annual Percentage Rate. The Effective Annual Rate basically takes into account the effect of compounding interests of the coupons.

1.6.2 Output Values

- Discount Rate per period (r) - Yield to Maturity is typically quoted like an Annual Percentage Rate. This discount rate is the exact rate per period. For example, if the Coupon Payment Frequency is semi-annually, then this discount rate is the rate per six months.
- Duration - Macaulay Duration. The composite measure of interest rate sensitivity of a bond. In the spreadsheet it is calculated as follows:

$$\text{Duration} = \text{Sum} ((\text{Present Value of Cash Flow at Time } t * t) / \text{Bond Market Price})$$

The Bond Market Price is calculated as the sum of the values in the column "PV of Cash Flows".

- Modified Duration - Adjusted Macaulay Duration. Also known as the Modified Duration. It is calculated as Macaulay Duration divided by 1 + yield to maturity.
- Convexity - The degree to which the duration changes when the yield to maturity changes. The column "(PV*(t^2+t))" is used for calculating the Convexity of the Bond. The formula for calculating bond convexity is shown below.

$$\text{Convexity} = (\text{Sum}(\text{PV}*(t^2+t)))/((1+\text{Discount Rate per period})^2)/\text{Bond Market Price}$$

1.7 **Estimating price change using the Modified Duration**

The "Using Modified Bond Duration" worksheet can be used for estimating the price change of a bond when there is a change in Yield. The price change is estimated using the Modified Duration.

	A	B	C	D	E	F
1	Estimating Price Change using Modified Bond Duration					
2						
3						
4	Coupon Payment Frequency*				Semi-Annually	
5	Yield to Maturity (Y)*					10.00%
6	Duration*					4.41
7	Percentage Change in Yield (Use minus for negative values)*					-0.75
8	Modified Duration					4.2
9	Percentage Change in the price of the Bond					3.15

1.7.1 Input Values

The following three input fields (which are explained in this document above) are used for calculating the Modified Duration:

- Coupon Payment Frequency
- Yield to Maturity (Y)
- Duration

The Percentage Change in Yield field is an input value for the calculation of the "Percentage Change in the price of the Bond".

1.7.2 Output Values

- Modified Duration - Adjusted Macaulay Duration. Also known as the Modified Duration. It is calculated as Macaulay Duration divided by $1 + \text{yield to maturity}$.
- Percentage Change in the price of the Bond - This is calculated as $-(\text{Modified Duration})/(1+Y) * (\text{Percentage Change in Yield})$.